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A reinvestigation has shown that – in contrast to the claim at the beginning of section 8 – apparently, the limit $r \downarrow 0$ and $\rho \downarrow 0$ in the relation (60) are **only** equivalent for the terms of order $O\big(\rho^{\pm B_j}\big)$, but are **not** equivalent for the terms of order $O(1)$. As a consequence, several factors of the form $\big(\sqrt{1-\lambda^2}/\sqrt{2}\big)^{\pm B_j}$ were missing in the computations that led to the equation (64). Denoting the correction factor by

$$D = \big(\sqrt{1-\lambda^2}/\sqrt{2}\big)^{B_j} \tag{1}$$

the terms $O(1)$ are computed to be:

$$\begin{aligned}
&-\frac{\sqrt{2}\sqrt{1+\lambda}}{(\kappa_j\sqrt{1-\lambda^2}+Z)(2\kappa_j+\sqrt{2}Z)}\Big\{D^{-1}C(-B_j,\lambda)\big((B_j-\kappa_j)\sqrt{1-\lambda^2}+(\lambda-1)Z\big)\\
&\times\big(C(B_j,-\mathrm{i})\sqrt{1+\mathrm{i}}(\sqrt{2}(B_j-\kappa_j)-(1-\mathrm{i})Z)+C(B_j,\mathrm{i})\sqrt{1-\mathrm{i}}(\sqrt{2}(B_j-\kappa_j)\\
&-(1+\mathrm{i})Z)e^{\mathrm{i}\theta}\big)+D\,C(B_j,\lambda)\big((B_j+\kappa_j)\sqrt{1-\lambda^2}+(1-\lambda)Z\big)\big(C(-B_j,-\mathrm{i})\sqrt{1+\mathrm{i}}\\
&\times(\sqrt{2}(B_j+\kappa_j)+(1-\mathrm{i})Z)+C(-B_j,\mathrm{i})\sqrt{1-\mathrm{i}}(\sqrt{2}(B_j+\kappa_j)+(1+\mathrm{i})Z)e^{\mathrm{i}Z}\big)\Big\}\\
&+\frac{\sqrt{2}\sqrt{1-\lambda}}{(\kappa_j\sqrt{1-\lambda^2}+Z)(2\kappa_j+\sqrt{2}Z)}\times\Big\{D^{-1}C(-B_j,\lambda)\big((B_j+\kappa_j)\sqrt{1-\lambda^2}+(\lambda+1)Z\big)\\
&\times\big(C(B_j,-\mathrm{i})\sqrt{1-\mathrm{i}}(\sqrt{2}(B_j+\kappa_j)+(1+\mathrm{i})Z)+C(B_j,\mathrm{i})\sqrt{1+\mathrm{i}}(\sqrt{2}(B_j+\kappa_j)+\\
&(1-\mathrm{i})Z)e^{\mathrm{i}\theta}\big)+D\,C(B_j,\lambda)\big((B_j-\kappa_j)\times\sqrt{1-\lambda^2}-(1+\lambda)Z\big)\big(C(-B_j,-\mathrm{i})\sqrt{1-\mathrm{i}}\\
&\times(\sqrt{2}(B_j-\kappa_j)-(1+\mathrm{i})Z)+C(-B_j,\mathrm{i})\sqrt{1+\mathrm{i}}(\sqrt{2}(B_j-\kappa_j)-(1-\mathrm{i})Z)e^{\mathrm{i}Z}\big)\Big\}
\end{aligned} \tag{2}$$

However, (2) is no longer in accord with the corresponding equation (16) of Ref.[27] (C. Burnap *et al*, Nuovo Cimento **64** 407), i.e., the terms $O(1)$ in (2) are only identical to the terms $O(1)$ given by Burnap *et al* in their equation (16) if $D = 1$.

Nonetheless, we do not see how to avoid these corrections. Whereas the right had side of the equation (64) is not affected by the corrections, the left hand side of (64) and the terms in (65) acquire a factor D^2 so that the corrected left hand side of (64) reads

$$- D^2 \frac{C[B_j, \lambda]\big((B_j - \kappa_j)\sqrt{1-\lambda^2} - (1+\lambda)Z\big)}{C[-B_j, \lambda]\big((B_j + \kappa_j)\sqrt{1-\lambda^2} + (1+\lambda)Z\big)} \tag{3}$$

while the corrected (65) becomes

$$- \Big(\frac{1-\lambda^2}{2}\Big)^{B_j} \frac{\Gamma(B_j - \lambda Z/\sqrt{1-\lambda^2})\big((B_j - \kappa_j)\sqrt{1-\lambda^2} - (1+\lambda)Z\big)}{\Gamma(-B_j - \lambda Z/\sqrt{1-\lambda^2})\big((B_j + \kappa_j)\sqrt{1-\lambda^2} + (1+\lambda)Z\big)}. \tag{4}$$

Analogous corrections have to be implemented in the corresponding equations (73) and (74) of section 9.

Unfortunately, these correction appear to invalidate (at least) several results given in the sections 10 and 11. In particular, if $Z^2 > \kappa_j^2$, for $\lambda \downarrow 0$ the additional factor D^2 in the corrected (65) is oscillating in such a way that it exactly cancels the oscillations of the other factors in (65), i.e., those oscillations that lead to the infinite number of λ for which the uncorrected (65) becomes equal to (66). Thus, theorem 10.1 appears to be incorrect and there is no acculmulation of the the discrete spectrum at the lower threshold -1 of the essential spectrum.

A detailed reinvestigation of the subject is in progress and more details will follow in due time.